



25

UNDP LAC
WORKING PAPER SERIES

**IMPLICATIONS
OF MINIMUM
WAGE POLICIES**

FOR LABOUR MARKETS WITH HIGH
INFORMALITY AND FRICTIONS

MARCH 2021

Luca Flabbi

University of North Carolina at Chapel Hill
United States

IMPLICATIONS OF MINIMUM WAGE POLICIES FOR LABOUR MARKETS WITH HIGH INFORMALITY AND FRICTIONS

Luca Flabbi¹

Abstract

This paper develops a theoretical framework to study the impact of minimum wage policies on labour markets with high informality. The model is an extension of a search and matching model where firms and workers form jobs that can be both formal and informal. Contrary to previous works on Latin America focusing on the impact of the minimum wage on inequality, the policy implications drawn from the model focus on overall welfare. The main result is that introducing or increasing a mandatory minimum wage has an ambiguous impact on workers' welfare. This ambiguity arises by taking into account not only the standard employment effects but also the equilibrium effects that change workers' bargaining power with respect to firms. Interestingly, the bargaining power impact is present both when bargaining for formal jobs that follow minimum wage rules and for informal jobs that do not.

Keywords: Minimum wage, Latin America, search and matching, bargaining, informality
 JEL codes: J24, J3, J64, O17

Background Paper for the UNDP LAC Regional Human Development Report 2021. The views and recommendations presented in this paper are from the authors and do not necessarily represent the official position of UNDP.

¹ Associate Professor. Department of Economics, University of North Carolina at Chapel Hill. United States. luca.flabbi@unc.edu.

Recommended citation: Flabbi, L. (2021). Implications of Minimum Wage Policies in Labor Markets with High Informality and Frictions. UNDP LAC Working Paper No. 25. Background Paper for the UNDP LAC Regional Human Development Report 2021.

1. Introduction

This background paper provides a framework to study the effects of minimum wage policies on labour markets characterized by high informality. The presence of high informality introduces additional channels in the transmission of any labour market policy to overall labour market allocations. In terms of the minimum wage, it is of particular importance to allow dynamic effects to occur. Does the presence of a minimum wage affect employment only in terms of the overall unemployment rate or does it also impact unemployment duration? Can the presence of dynamic effects partially offset the negative employment impact by generating better matches of workers to firms? When the outside option of formal employment is not simply unemployment but also informal employment, are the effects of a minimum wage policy reduced or magnified?

To answer these policy-relevant questions, this paper extends the most popular framework used to model labour market dynamics: search and matching models.¹ This framework is particularly appropriate to study the impact of minimum wage policies because it provides a tractable characterization of the dynamics and allows for the non-competitive features ('frictions') that may justify policy interventions.² Search and matching models have been used to study labour markets with high informality but without a specific focus on minimum wage policies.³

In fact, this paper shows that the impacts of minimum wages policies depend on the labour market structure and the model used to analyse it. In a perfectly competitive model, the effects are well understood; however, the results may be misleading if the labour market under consideration is not well approximated by a competitive model. Given the institutional features of typical Latin American labour markets, this paper argues that the competitive model is not a good approximation and that a model with frictions can better capture their crucial characteristics. Since search costs and frictions generate inefficiencies, there is room for policy interventions, including a minimum wage policy, to improve welfare.⁴

The primary policy objective of a mandatory minimum wage (i.e. a 'price floor' on the factor of production labour) is to transfer resources to workers in low-paying jobs. In a static setting and in a competitive labour market, the policy's effect is intuitive: some low wage workers will earn more while others may lose their jobs (the 'employment effect') because labour is now more expensive. The extent of employment effects is a function of the market structure. Both the level of market competition and the type of outside options available to workers and firms are relevant to characterize market structure. Typical outside options are unemployment and non-participation. If a large informal sector is present, working in an informal job becomes an additional outside option for both workers and firms since, by definition, it is a portion of the labour market where the mandatory minimum wage is not enforced.

¹ For a review of the applied literature, see Eckstein and van den Berg (2007); for a review of the macro and theory literature, see Rogerson et al. (2005).

² For these reasons, numerous contributions have used a search and matching framework to study minimum wage policies. They include Eckstein et al. (2011), Flinn (2006, 2011) and van den Berg and Ridder (1998).

³ For applications in Brazil, see Bosch and Esteban-Pretel (2012) and Meghir et al. (2015); for applications in Mexico, see Bobba et al. (2017).

⁴ While the result depends on parameters, it opens the door to the possibility of optimal policy. This is one of the crucial messages from Flinn (2006, 2011).

In a dynamic setting, additional channels come into play. First, the employment effects are reflected not only in overall levels of unemployment but also in unemployment duration. If unemployment durations increase, additional negative impacts may be felt, such as human capital depreciation or ‘discouraged worker’ effects. If workers spend too much time unemployed, they may give up searching altogether (becoming ‘discouraged’) and leave the labour force. This may make their re-entry in the labour market more difficult and may exacerbate their human capital depreciation. Second, in a market where movement among jobs is costly and information about job availability is imperfect, dynamic effects offer an opportunity. By searching longer, workers and firms may make better matches. Third, if a bargaining structure is present, the presence or increase of a mandatory minimum wage may increase the workers’ bargaining power and generate ‘spillover effects’. Spillover effects occur when workers earning above the minimum wage level also see their wages increase owing to the presence of or increase in the minimum wage.

The crucial question that must be answered when evaluating minimum wage policies is: if the policy transfers resources to workers in low-paying jobs, from whom are these resources extracted? In a static and competitive setting, firms hire fewer workers, implying a transfer of resources among low-earning workers. In a dynamic setting characterized by search and bargaining, some resources are transferred from the firms themselves because the minimum wage affects how rents are split (Flinn, 2011). When a large informal setting is present, this latter channel may be greatly reduced.

This variety of factors leads to the variety of effects found when evaluating minimum wage policies. The literature on Latin America has focused primarily on the impact of minimum wage policies on inequality, rather than their impact on overall welfare or optimal policies that target inefficiencies. When measured over labour income and over the short and medium term, the impact on inequality seems quite clear: the presence of a binding minimum wage reduces inequality. For example, Bosch and Manacorda (2010) conclude that lowering the real minimum wage in Mexico between the late 1980s and the early 2000s was largely responsible for the increase in inequality in labour market income over that period. Engbom and Moser (2018) find the complementary effect in Brazil: the increase in real minimum wage between the late 1990s and the late 2000s seems to have had a significant impact on reducing inequality and poverty. Contrary to these seemingly unambiguous results in terms of inequality, this background paper found that introducing or increasing a mandatory minimum wage had an ambiguous impact on workers’ welfare. The result is obtained by taking into account not only the standard employment effects but also the equilibrium effects that change workers’ bargaining power with respect to firms. Interestingly, the impact on bargaining power is present both when bargaining for formal jobs that follow minimum wage rules and for informal jobs that do not.

The remainder of this paper is organized as follows: section 2 presents the theoretical framework, introducing the formal model in sections 2.1-2.4 and providing a more intuitive discussion and comparison with the competitive model in section 2.5. Section 3 is devoted to policy implications, including a schematic representation discussed in section 3.1. Section 4 provides a summary and conclusion.

2. Model

2.1. Environment

This paper proposes an environment in which the labour market is characterized by frictions, workers and firms make decisions based on expectations about the future and wages are determined by bargaining. There are four labour market states: unemployment, self-employment, informal salaried employment and formal salaried employment.

Agents search for a salaried position and for self-employment opportunities. A meeting between a potential salaried worker and a firm produces a match-specific productivity with monetary value $x \sim G_k(x)$, where $k = 0, 1$ denotes an informal or a formal match, respectively. Based on productivity and formality status, workers and firms engage in bargaining to determine the wage $w_k(x)$. At the end of the process, they decide whether to accept the match. A searcher who finds a self-employment opportunity also decides whether to accept. The flow value of such an opportunity is the net productivity $y \sim R(y)$.⁵

The formality status has an impact on both the cost side and the benefit side of the labour relationship. Specifically, the differences between working as a formal or informal salaried worker are as follows:

- **Costs:** Working formally implies compliance with labour legislation, which is described here with two parameters: a proportional payroll contribution rate t and a mandatory minimum wage w_M . This model assumes that the payroll contribution is withdrawn at the source by firms and that there is full compliance with the minimum wage mandate. Analytically, full compliance with the minimum wage is implemented by introducing a constraint when workers and firms bargain for wages. Working informally implies the risk of being audited and fined. This cost is parametrized with a parameter c , without attempting to separate the probability of being audited and the amount of the fine. Consistent with much of the legislation in the region of Latin America and the Caribbean, the cost is assumed to be paid by the firm. Of course, given the bargaining structure, it can be partially transferred to the workers, much like the payroll tax. It is interesting to note that even if this enforcement cost were extremely high, informality would not disappear. A high enough c could eliminate the presence of informal salaried workers in equilibrium, but it would have only a limited impact on the presence of the other informal state: self-employment. This other informal state is typically sustained by institutions that exclude self-employed workers from the obligation to contribute.⁶
- **Benefits:** In addition to wages, jobs provide social security benefits such as health, housing or retirement benefits. However, these benefits differ depending on whether the job is formal or informal. Specifically, an informal job cannot provide a better benefit than that received by unemployed or self-employed individuals. These benefits are denoted with $\beta_0 B_0$, distinguishing between the valuation of the benefit β_0 (a preference parameter) and the monetary value of the benefit B_0 (the government expenses to provide the benefit). In contrast, a formal job can provide a different benefit, since

⁵ I.e. any set-up cost or flow costs are abstracted.

⁶ This is the institutional setting of some Latin American countries, including Chile, the Dominican Republic, Honduras, Mexico, Peru and, in part, Colombia.

formal salaried workers contribute to their social security benefits through the payroll contribution rate t . This benefit is denoted with $\beta_1 B_1$. In many institutional frameworks, at least a portion of this benefit is a function of the contribution. A typical example is retirement benefits, which increase with the amount contributed by firms and the worker. The theoretical model presented here ignores this dependence to avoid complications and better focus on minimum wage issues. Nevertheless, such institutional nuance can be analysed within a framework similar to the one presented here. (See for example Bobba et al., 2017, 2020)

The self-employed state shares some aspects of informal jobs and some aspects of formal jobs. A large portion of self-employed workers in Latin America and the Caribbean work in activities that can be described as a ‘residual’ labour market state (Fields, 1975), such as selling refreshments on street corners or establishing microenterprises in their homes. These types of activities do not require significant set-up costs (Bianchi & Bobba, 2013; McKenzie & Woodruff, 2006) and are therefore comparable to searching for a salaried position. They are also activities for which agents pay very few contributions. In some Latin American and Caribbean countries, such as Mexico, agents are not legally required to make contributions and, in fact, avoid them completely. For this large share of the self-employed labour force, it is reasonable to assume that agents do not to pay social security contributions and therefore receive the same non-contributory benefit received by informal salaried workers. This assumption is adopted in the model and implies that the informal sector is composed of two types of workers: informal salaried workers and self-employed workers.

Another portion of the self-employed labour force is significantly different, made up of skilled agents working in their own professional activities, such as consultants and medical professionals. It also includes entrepreneurs with small- and medium-sized enterprises with a number of salaried workers. This type of self-employment activity involves a relatively small number of agents and is concentrated in the most skilled strata of the labour force. Since this portion of the labour force is not the most likely to be affected by minimum wage regulations, the model will ignore this form of self-employment and concentrate on the ‘residual’ type of self-employment described above.

To summarize, the model makes the following assumptions:

- Technology determines the distribution of match-specific productivity $G_k(x)$ and the distribution of self-employment income $R(y)$.
- The labour market structure determines the extent of the frictions where labour supply meets labour demand.
- Institutions determine policy parameters, including the mandatory minimum wage w_M , the cost of hiring informally c and the features of the social security system $\{t, \beta_k B_k\}$.
- Workers and firms endogenously determine wage distributions, employment level and informality level.

Before proceeding to the formal description of the model, it is useful to introduce some notation. Table 1 reports each labour market state with the four elements characterizing the states: the present discounted value, the steady state equilibrium proportion, the shocks and the flow utility. In the first row, individuals in the unemployment state are looking for jobs and receive offers at Poisson rate λ . This is the only idiosyncratic shock to which they are subject. While searching, they receive a flow utility ζ and flow benefit B_0 , which they value at β_0 per unit of currency. The flow utility ζ can be either positive or negative since it is the net benefit

of being in the state. It is positive if the search cost and other utility costs related to unemployment are lower than the leisure time they could enjoy in the state; otherwise, it is negative. The second row represents the self-employment state: workers in this state are subject to a termination shock at Poisson rate δ and receive flow utility equal to the self-employment income y plus the same non-contributory benefit received by the unemployed. A termination shock is a shock that destroys the job and forces the agent to return to the searching state in order to find a new job. The third and fourth rows represent the informal and formal salaried state, respectively. They are both subject to a termination shock at Poisson rate η_k and their flow utility is composed of the wage plus benefits. The difference is in the wage schedule and in the level and evaluation of benefits. At the same match-specific productivity x , the bargaining process may generate a different wage depending on formality status (see section 2.3). In addition, a formal salaried worker will receive the contributory benefit B_f , while an informal salaried worker will receive the same non-contributory benefit as the self-employed and the unemployed.

Table 1: Model notation

State	Value function	Proportion	Shocks	Flow utility
Unemployed	U	u	λ	$\xi + \beta_0 B_0$
Self-employed	$S[y]$	s	δ	$y + \beta_0 B_0$
Informal salaried worker	$E_0[w_0(x)]$	e_0	η_0	$w_0(x) + \beta_0 B_0$
Formal salaried worker	$E_1[w_1(x)]$	e_1	η_1	$w_1(x) + \beta_1 B_1$

Source: Author.

Note: The four labour market states are mutually exclusive and cover all labour market participants. As a result, $u + s + e_0 + e_1 = 1$, the unemployment rate is u , and the informality rate is $(s + e_0)$.

2.2. Value functions

Given the environment, it is possible to write the present discounted value of being in each labour market state in recursive form. Each value has the usual interpretation of the sum between the flow value and the continuation value.

The value of an unemployed worker is determined by the following expression:

$$\begin{aligned}
 (\rho + \lambda)U &= \xi + \beta_0 B_0 \\
 &+ \lambda \left\{ \begin{array}{l} \pi_1 \int \max [E_1[w_1(x)], U] dG_1(x) \\ + \pi_0 \int \max [E_0[w_0(x)], U] dG_0(x) \\ + (1 - \pi_1 - \pi_0) \int \max [S[y], U] dR(y) \end{array} \right\}
 \end{aligned} \tag{1}$$

The left-hand side of equation 1 is the value of unemployment U subject to the ‘effective’ discount rate $(\rho + \lambda)$. The first row of the right-hand side of equation 1 is the flow value: the (dis) utility from unemployment ξ plus the utility from non-contributory benefits $\beta_0 B_0$. The second row is the innovation part of the continuation value. If the worker finds a job offer—an event occurring at rate λ —the offer can be as a formal salaried worker with probability π_f , as an informal salaried worker with probability π_0 or as a self-employed individual with the residual probability. The productivity (either x or y) of each of these three job offers is not known *ex ante* but only revealed when the meeting occurs, which is why the continuation value is

subject to the expectation over $G_1(x)$, $G_0(x)$ and $R(y)$, respectively. Once the offer is found, the agent decides whether to accept it by solving a simple maximization problem between the value of the offer and the value of the status quo U . It is interesting to note how the value depends on the probability of receiving an offer π_0 and π_1 . The higher these probabilities, the higher the value of searching U . Receiving more offers has an unambiguously positive impact because workers can always reject the offers they do not like. In terms of the impact of the minimum wage, it is crucially important for π_1 ; the higher the probability of finding formal salaried offers, the more important the minimum wage since it applies only to that type of offer. Finally, in a full equilibrium model, these probabilities are endogenous; they are a function of the supply (workers searching for jobs) and the demand (firms searching to fill vacancies) in the market. For tractability, the model presented in this paper does not explicitly allow for endogenous contact rates, but a large body of literature exists in which this endogeneity is introduced within a similar search framework.⁷ It is worth emphasizing that the presence of endogenous contact rates could both magnify and reduce the impact of minimum wage policies. There are no applications involving minimum wages, informality and endogenous contact rates; however, Flinn and Mullins (2015) employ an application in which both minimum wages and endogenous contact rates are present, in addition to formal schooling decisions. They find that the optimal minimum wage is slightly higher when the contact rates are endogenous than when they are kept fixed, as in the framework presented here. On the other hand, without an endogenous schooling decision, Flinn (2006) finds the opposite: the optimal minimum wage with exogenous meeting rates is more than twice the optimal minimum wage with endogenous meeting rates.

The value of an informal salaried worker in a match with productivity x is determined by the following expression:

$$\begin{aligned}
 (\rho + \eta_0)E_0[w_0(x)] &= w_0(x) + \beta_0 B_0 \\
 &+ \eta_0 U
 \end{aligned} \tag{2}$$

Again, the left-hand side reports the value of the state, subject to the effective discount rate. The first row of the right-hand side is the flow value: the wage $w_0(x)$ plus the same non-contributory benefit received by the unemployed. The innovation part of the continuation value includes only the value of searching as an unemployed individual. The agent will be in this state when receiving the termination rate, an event occurring a rate η_0 .

The value of a formal salaried worker in a match with productivity x is similar, determined by the following expression:

$$\begin{aligned}
 (\rho + \eta_1)E_1[w_1(x)] &= w_1(x) + \beta_1 B_1 \\
 &+ \eta_1 U
 \end{aligned} \tag{3}$$

⁷ See Petrongolo and Pissarides (2001) for a review on the most common feature used to introduce endogenous contact rates: the matching function. For applications with endogenous contact rates and informality (but no minimum wage), see Bobba et al. (2017) and Bosch and Esteban-Pretel (2012).

All the elements composing equation 3 have a similar interpretation as those in equation 2. Notice that this is the only state receiving the contributory benefit $\beta_1 B_1$.

Finally, the value of a self-employed individual in a job with productivity y is determined by the following expression:

$$\begin{aligned}
 (\rho + \delta)S[y] &= y + \beta_0 B_0 & (4) \\
 &+ \delta U
 \end{aligned}$$

Again, the interpretation is similar to the previous expressions. The primary difference is in the flow value, for which self-employed agents receive the entire productivity y .

2.3. Wage determination

As mentioned, wages are determined by bargaining. A common and tractable way to solve such bargaining problems is to assume the axiomatic Nash bargaining solution.⁸ This procedure assigns the wage at a value that maximizes the sum of the surplus of both agents involved in the negotiations: the worker and the firm. The worker's surplus is simply the difference between the new value if the match is made $E_k[w_k(x)]$ and the status quo value the agent brings to the negotiation U . The firm's surplus is assumed to be the discounted value of the flow profits. Some foundation for this value can be given by assuming that firms are free to enter the market, in which case, they would do so until the value of posting a vacancy is zero. Under this scenario, the difference between the value of a filled position and the value of a vacancy is exactly equal to the stream of flow profits appropriately discounted. In conclusion, this provides:

$$w_1(x) = \operatorname{argmax} [E_1[w_1(x)] - U]^\alpha \left[\frac{x - (1+t)w_1(x)}{(\rho + \eta_1)} \right]^{(1-\alpha)} \quad (5)$$

$$w_0(x) = \operatorname{argmax} [E_0[w_0(x)] - U]^\alpha \left[\frac{x - w_0(x) - c}{(\rho + \eta_0)} \right]^{(1-\alpha)} \quad (6)$$

where α is a measure of the bargaining power of the worker with respect to the firm. If $\alpha = 0.5$, there is 'symmetric' bargaining; if $\alpha < 0.5$, the firm is in a relatively strong bargaining position; if $\alpha > 0.5$, the worker is in a relatively strong bargaining position.

The solution to this surplus-maximizing problem generates the following one-to-one mapping between wages and match-specific productivity:

$$w_1(x) = \alpha \frac{x}{1+t} + (1-\alpha)[\rho U - \beta_1 B_1] \quad (7)$$

$$w_0(x) = \alpha(x - c) + (1-\alpha)[\rho U - \beta_0 B_0] \quad (8)$$

⁸ See, for example, Bobba et al. (2017), Eckstein and Wolpin (1995) and Flinn (2006).

The structure of the wage schedule for both formal and informal workers is similar. The wage is a combination of the match-specific productivity x and the worker's outside option ρU . The first element introduces a positive relationship between productivity and wages. The second element introduces a direct link between the worker's search behaviour and outside option and the wage: the higher the worker's outside option, the larger the share of the surplus going to the worker and the higher the wage. Match-specific productivity x and the worker's outside option ρU are weighted in the wage schedule by the parameter α : the higher α , the higher the relative bargaining position of the worker and the more weight is given to productivity in determining the wage. In addition to these fundamental elements, the wage equation also includes institutional parameters: the benefits (contributory and non-contributory) and the costs (payroll contribution or the cost of being caught hiring informally). Both the payroll contribution t and the informality cost c are paid by the firm; however, through bargaining, the firm is able to share some of that cost with the worker, i.e. the higher t or c , the lower the wage. By the same reasoning, the presence of benefits allows firms to pay the worker less for a given level of productivity x (notice that both $\beta_k B_k$ are inversely proportional to wages).

This structure clarifies how the departure from the competitive model has a direct impact on the price of labour. In a competitive model, wages are equal to marginal productivity. In this setting, the condition would be equivalent to $w_k(x) = x$ (ignoring c and t). In a search model, wages are lower than marginal productivity. Again, ignoring the specific institutional context, wages take the form $w_k(x) = \alpha x + (1 - \alpha)\rho U$. This equation always delivers an accepted wage lower than marginal productivity for value of α between 0 and 1 because the reservation wage is equal to ρU . The economic source of this result is the presence of market power induced by frictions; both the worker and the firm have some market power because both face a cost by rejecting the match and returning to the search. In other words, any match generates a surplus that could then be divided between the firm and the worker. For given outside options, α is exactly the parameter that regulates how the division is taking place. As long as its value is between 0 and 1, each side of the market receives a portion of the surplus. For the two extreme values of α (0 and 1), one side of the market receives all the surplus. If $\alpha = 1$, all the bargaining weight is given to the workers, and wages are equal to marginal productivity under perfect competition. If $\alpha = 0$, all the bargaining weight is given to the firms, and wages are equal to the reservation wage for all workers, independent of productivity. These same results carry through in the specific institutional setting analysed here, with a few differences. First, the surplus is reduced by the presence of payroll contributions t and the cost of informality c , and second, the benefits impact the value of the workers' outside options.

In the presence of a mandatory minimum wage w_M , agents solve the maximization problem (equation 5) under the constraint that $w_1(x) \geq w_M$. In equilibrium, this constraint generates a corner solution with wages equal to the minimum wage for relatively low values of productivity. For relatively high values of productivity, wages follow a wage schedule similar to equation 7. Even more interesting, the presence of the minimum wage generates **equilibrium** effects since it does impact the overall value of participating in the market U . This issue will be discussed in more detail in section 2.4, but the economic intuition can be anticipated here. Since wages at **both** formal and informal salaried jobs are a function of the outside option ρU and the introduction of a mandatory minimum wage w_M impacts the outside option, then **all** wages in the economy are affected by w_M , not simply the wages of workers earning the minimum wage in formal jobs. Suppose that the introduction of the minimum wage increases the value of participating in the market. Then all formal wages above the minimum

wage will increase, a phenomenon that is well known in the literature.⁹ In addition, all informal wages will also increase (at same value of productivity), even though the informal sector does not comply with the minimum wage regulation. This phenomenon is less common in the literature, and its interpretation through the value of the outside option is novel to the approach proposed in this paper.

2.4. Equilibrium

Searchers must decide whether to accept a match with a firm or a self-employment opportunity. If they reject it, they will continue searching. Equation 1 shows that the value of rejection is a constant equal to U , while equations 2 and 3 show that the value of accepting a salaried job offer is increasing in x and equation 4 shows that the value of accepting a self-employment opportunity is increasing in y . As a result, for extremely low values of x and y , searchers will prefer to continue searching, while for sufficiently high values of x and y , they will prefer to accept the offer. Critical values (the ‘reservation’ values) will therefore exist at which the agent is indifferent. This reservation value property of the optimal decision rules provides a simple description of the optimal decision rules in this rather complex dynamic optimization problem.

2.4.1. Equilibrium without minimum wage

It is easier to begin with the case **without** minimum wage, for which the reservation values are defined as:

$$E_1[w_1(x_1^*)] = U \Leftrightarrow w_1^* = \rho U - \beta_1 B_1 \Leftrightarrow x_1^* = [\rho U - \beta_1 B_1](1 + t) \quad (9)$$

$$E_0[w_0(x_0^*)] = U \Leftrightarrow w_0^* = \rho U - \beta_0 B_0 \Leftrightarrow x_0^* = [\rho U - \beta_0 B_0] + c \quad (10)$$

$$S[y^*] = U \Leftrightarrow y^* = \rho U - \beta_0 B_0 \quad (11)$$

The optimal decision rules state simply that a searcher should accept the match when the productivity is higher than the reservation value and should continue searching otherwise. The reservation values have an intuitive form: the reservation wages must compensate for the outside option ρU and the benefit $\beta_k B_k$. The reservation productivity values must additionally compensate for the firm’s cost to hire formally or informally, described by the parameters t and c , respectively. Both costs increase the critical values since they decrease the match’s surplus, as shown in the last expressions of equations 9 and 10.

To complete the definition of the equilibrium, consider that the optimal decision rules described can be inserted in the value functions of equations 9 and 10, leading to expressions

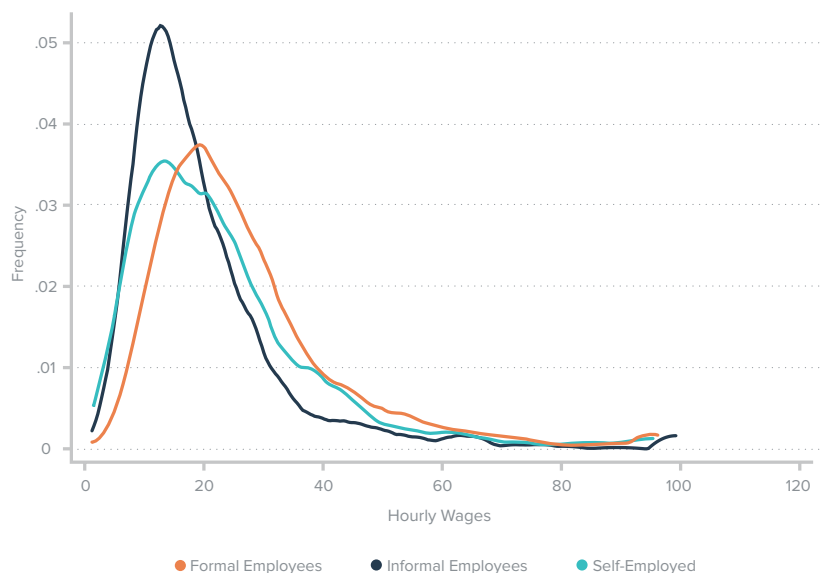
⁹ See for example, Card and Krueger (2015) and Flinn (2011).

that are only a function of parameters and reservation values. But since the reservation values are only a function of parameters and the value of unemployment U , the equilibrium can be formally solved as a fixed point in U . Given the regularity of the environment, only mild conditions on the parameters are necessary to deliver existence and uniqueness. Notice also that the transitions between labour market states and, therefore, the laws of motion governing the proportions in each state are fully characterized by the optimal decision rules together with the Poisson arrival rates. By imposing steady state conditions on those laws of motion, the equilibrium proportion of unemployment, formal and informal employment and self-employment can be fully defined.

The optimal decision rules map directly into observable labour market outcomes: accepted wage distributions, self-employment income distributions made and labour market states. Accepted wage distributions are the distributions obtained by applying the wage schedules from equations 7 and 8 to the productivity in salaried jobs $G_1(x)$ and $G_0(x)$, **truncated** at the reservation values x_0^* and x_j^* . The same truncation occurs on the productivity in self-employment $R(y)$ but at the value y^* . Notice that these optimal decision rules are quite flexible in generating a variety of rankings and shapes of observed wage and income distributions. The only constraint imposed is that wages and income should be positive, since positive reservation values exist, as shown in equations 9-11. Based on this, different primitive distributions $G_1(x)$, $G_0(x)$ and $R(y)$ can generate different rankings between formal wages, informal wages and self-employment income. Crucially, the model can generate a significant amount of **overlap** between the labour income distributions of these three labour market states. To illustrate, figure 1 reports the observed empirical densities of a sample of workers in Mexico.¹⁰ The distributions are representative of other countries in the region: formal salaried workers earn more on average than informal salaried workers, while the self-employed earn on average an amount between the two salaried worker states. Equally common is the significant amount of overlap between the three distributions: many informal salaried workers earn more than formal salaried workers, and self-employed individuals earn both more and less than both types of salaried workers. The model's flexibility allows it to fully replicate density distributions such as the ones reported in figure 1.

¹⁰ The data are extracted from the 2005 Encuesta Nacional de Ocupación y Empleo [National Survey on Occupation and Employment] and elaborated by Bobba et al. (2017). See the footnote to the figure for additional definitions.

Figure 1: Labour income density functions



Source: 2005 Encuesta Nacional de Ocupación y Empleo [National Survey on Occupation and Employment], as presented in Bobba, M., Flabbi, L., & Levy Algazi, S. (2017). Labor market search, informality and schooling investments (Discussion paper No. 11170). IZA Institute of Labor Economics.

Note: The figure shows the empirical densities of the hourly earnings for non-agricultural male salaried workers and self-employed individuals between the ages of 25 and 55 years who reside in urban areas and completed secondary schooling. Earnings are reported in Mexican pesos. The formal/informal status of the job is defined according to whether workers report having access to health care through their employers.

2.4.2. Equilibrium with minimum wage

When a mandatory minimum wage is introduced, the decision to accept a formal job is more complicated but still tractable, as shown in Flinn (2006, 2011). This model defines $U(w_M)$ as the value of unemployment in a labour market with mandatory minimum wage w_M . It also assumes that the minimum wage is binding, that is $w_M > w_j^*$. In this economy, the broad labour market dynamic is similar to that without a minimum wage. Wage schedules are determined by bargaining, value functions are defined as in equations 1-4 and the equilibrium is characterized by reservation value rules. For example, the equation defining the unemployment state $U(w_M)$ will be exactly equal to equation 1, with $U(w_M)$ in place of U . The differences occur in the optimal decision rules and the constraints that must be considered when bargaining for wages.

Consider a worker finding a formal job with productivity well above the reservation value x_j^* , so that the corresponding wage is well above the mandatory minimum wage. In this case, the minimum wage constraint while bargaining does not bind, and the wage will simply be:

$$w_1(x; w_M) = \alpha \frac{x}{1+t} + (1-\alpha)[\rho U(w_M) - \beta_1 B_1] \tag{12}$$

The only difference in this wage schedule with respect to equation 7 is in the outside option $U(w_M)$. If $U(w_M) > U$, then this worker receives a higher wage than he or she would have received in a market without minimum wage. This is the first equilibrium effect described in section 2.3: formal wages **above** the minimum wage increase.

Now focus on a worker finding an informal job opportunity with productivity above the reservation value. Since the job is informal, there is no compliance with the minimum wage and the bargaining problem is therefore the same as described in equation 6. As a result, the wage schedule will be:

$$w_0(x; w_M) = \alpha(x - c) + (1 - \alpha)[\rho U(w_M) - \beta_0 B_0] \quad (13)$$

If $U(w_M) > U$, then this informal worker also receives a higher wage than he or she would have received in a market without minimum wage. This is the novel equilibrium effect described in section 2.3: even workers in informal jobs will receive a wage increase as a result of introducing a mandatory (and binding) minimum wage. The only labour market state without impact on earnings is self-employment. The compensation received by a self-employed individual remains equal to y , the productivity of the self-employment opportunity. This dynamic creates an ambiguous impact on inequality: salaried workers' wages increase while self-employed income is unaffected. The overall level of inequality increases or decreases depending on the proportion of workers that remain self-employed or unemployed in equilibrium. These considerations also indicate that cross-sectional inequality may be significantly different from dynamic and longitudinal inequality, i.e. inequality based on observing a snapshot of the labour market at a given point in time may be significantly different from inequality evaluated on the entire labour market career of each worker participating in the labour market.¹¹

There is one final important impact of the minimum wage left to be analysed: what happens to matches in the formal sector that are so close to the reservation value that they risk not being profitable if the minimum wage is paid? Consider first the wage equation 12. There will exist a productivity level \bar{x}_M such that the optimal wage is exactly equal to the minimum wage:

$$\begin{aligned} \bar{x}_M : w_1(\bar{x}_M; w_M) &= w_M & (14) \\ &\Leftrightarrow \\ \bar{x}_M &= \frac{(1+t)}{\alpha} \{w_M - (1-\alpha)[\rho U(w_M) - \beta_1 B_1]\} \end{aligned}$$

For all $x \geq \bar{x}_M$, the match is made and the wage obeys equation 12. For $x < \bar{x}_M$, firms would prefer to pay $w_1(x) < w_M$, but they are forced to pay w_M . When will it be optimal to do so? Only when profits are positive, i.e. as long as $[x - (1+t)w_M] \geq 0$. In such cases, the firm is willing to give the worker a larger share of the surplus to avoid losing a profitable opportunity. However, if the match-specific productivity is so low that paying the minimum wage delivers negative profits, the firm would prefer to give up the match and keep the vacancy open. There will then exist another critical productivity level \underline{x}_M , such that a wage equal to the minimum wage generates zero flow profits:

$$\begin{aligned} \underline{x}_M : \underline{x}_M - (1+t)w_M &= 0 & (15) \\ &\Leftrightarrow \\ \underline{x}_M &= (1+t)w_M \end{aligned}$$

¹¹ Flinn (2002) points out this difference in comparing the United States and Italy. He concludes that, while cross-sectional inequality is higher in the United States, inequality based on longitudinal measures is higher in Italy. Flabbi and Leonardi (2010) confirm that the difference is relevant, by comparing inequality in the United States over time.

A complete picture of the optimal decision rules in the presence of a minimum wage had now been presented. If the match-specific productivity is too low ($x < \underline{x}_M$), the match is rejected. If the match-specific productivity is high enough ($\bar{x}_M \leq x$), the match is made and the worker is paid according to the wage equation $w_1(x; w_M)$, defined in equation 12. If the match-specific productivity assumes an intermediate value ($\underline{x}_M \leq x < \bar{x}_M$), the match is made and the worker is paid the mandatory minimum wage w_M .

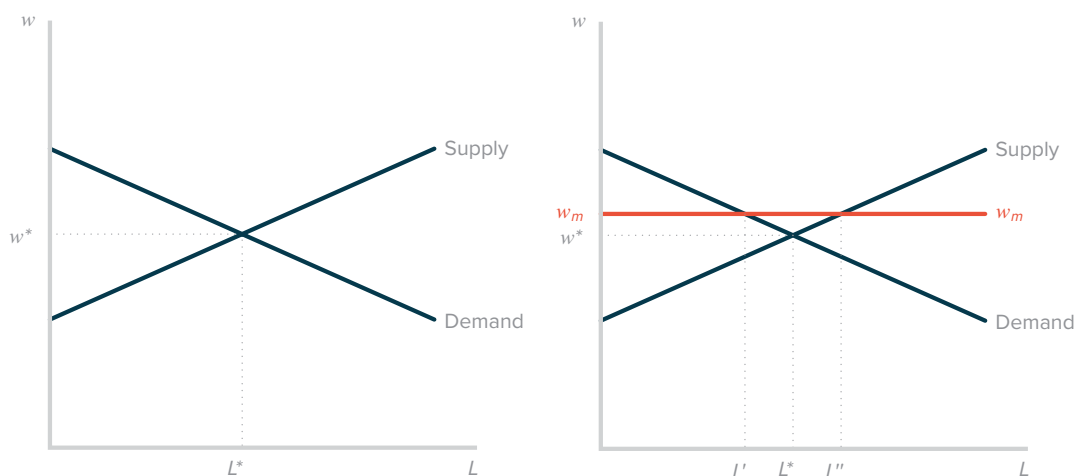
2.5. Discussion

It may be useful to contrast the labour market equilibrium defined above with the labour market with minimum wage typically analysed in the literature.

2.5.1. Comparison with the competitive model

Figure 2 shows a standard representation of supply and demand in a static competitive market. The top panel shows the competitive equilibrium; the bottom panel shows the equilibrium in the presence of a binding minimum wage. The minimum wage is binding because it is set at a value above the equilibrium wage w^* . When the minimum wage is binding, firms are willing to hire a lower quantity of labour ($L^1 < L^*$), even if the quantity of workers willing to supply labour at that wage is higher ($L^1 > L^*$). As a result, involuntary unemployment equal to $L^1 - L^*$ arises in equilibrium. In addition, the overall level of employment is lower than in the pre-policy environment (with a net loss equal to $L^* - L^1$), while the wage earned by employed workers is higher than in the pre-policy environment (since $w_M > w^*$). This is the **standard** employment effect: there is a loss of employment in exchange for a wage gain.

Figure 2: Minimum wage in a competitive model



Source: Author.

The search model of the labour market presented in section 2.1 is different from the static competitive model in three main dimensions,¹² which significantly affect prices and allocations in the labour market. In general, wages will **not** be equal to marginal productivity, and allocations will be affected by significant transaction costs.

First, finding a job is **not** a costless or frictionless process. Workers are not immediately informed of all available jobs, and firms are not able to screen all available workers instantly and at no cost. These costs and frictions are captured in the model by the Poisson process with parameter λ . Workers and firms meet at intervals of different lengths: the higher the λ , the shorter the average interval and the closer the labour market is to perfect competition.

Second, since the search process is costly, some **rents** are generated when workers and firms meet. The source of the rents is the search process itself: each agent knows that the other will face a cost to generate another match. The model presented in section 2.1 proposes the use of Nash bargaining to share these rents. In this setting, the rents are shared between the worker and the firm in proportion to their respective bargaining power. Each agent's bargaining power is a function of two elements: the value of the outside option and the Nash bargaining coefficient. The value of the outside option plays a role because it is the state to which each agent will return if the match is not made. The higher the outside option, the higher the share a given agent will receive. The outside options in the model are straightforward: they are the value of continuing to search. The worker's value of continuing to search is positive and equal to U ; the firm's value is equal to zero as a result of the 'free-entry' assumption, which posits that firms will continue to enter the market as long as doing so guarantees positive expected profits. The less likely this assumption is, the higher the firm's outside option and the higher the share it will receive.¹³ In addition to the outside options, other factors may influence rent sharing. These other factors are summarized by the Nash bargaining coefficient α (see equations 5 and 6). If $\alpha = 1$, the worker is able to extract the highest share, since the wage will be equal to the match-specific productivity x . If $\alpha = 0$, the firm is able to extract the highest share, since the wage will be exactly equal to the worker's outside option ρU . Both results are specializations of wage equations 7 and 8.

Third, dynamic considerations are important and cannot be captured by a static setting. The search process, rent sharing and termination shocks demonstrate the importance of the dynamic and option values, which constitute the final difference with respect to a standard static competitive model. Agents' decisions take into account not only contemporaneous utility but also the option value given up by taking a certain action. In the context of the model presented, workers decide on job offers not only based on the wage received but also on the fact that accepting the wage will reduce or eliminate the possibility of receiving a higher wage in the future. Firms make similar considerations; filling a vacancy with a worker generating a given productivity x prevents a firm from finding a potentially more productive worker to fill the same vacancy.

The differences between this proposed search model of the labour market and the standard static competitive model can have significant implications when studying the impact of the minimum wage on labour market outcomes. While some effects are qualitatively similar to

¹² For a good comparison of the search model of the labour market with a perfectly competitive model and the historical reasons for its development, see Mortensen's 2010 Nobel lecture (Mortensen, 2011).

¹³ In the context of an economy with high informality, this assumption may be more plausible for small firms that primarily hire informally than for bigger firms that face high entry costs and primarily hire formally.

the standard employment effects described above, their magnitude can differ significantly. Other effects are new and may lead to vastly different policy implications.

2.5.2. Comparison with the search friction model without informality

Figure 3 shows how these baseline impacts change in a model with frictions where the labour market dynamic is fully characterized. This is the environment analysed by Flinn (2006, 2011). The top panel represents the equilibrium without minimum wage in the space of productivity x and wages w . The line represents the wage schedule determined by Nash bargaining, and the starred values are the relevant reservation thresholds. Any match-specific productivity higher than x_j^* is made, since both the worker and the firm agree that the match is better than the alternative. In equilibrium, the wage is determined by the wage line $w_1(x)$,¹⁴ and the level of employment is determined in the steady state in the same way as the one discussed in section 2.4. Notice that the proportion of workers employed is decreasing in x_j^* : the higher the reservation productivity value x_j^* , the lower the employment rate.

The middle panel in figure 3 depicts an intermediate step between equilibrium **without** minimum wage and equilibrium **with** minimum wage. This intermediate step simplifies the analysis because it does not take into account the equilibrium effect, i.e. the impact of the minimum wage on workers' outside options. Ignoring the equilibrium effect, the wage schedule is the same as that in the top panel; however, if the minimum wage is binding, firms cannot pay a wage as low as w_j^* and must pay a minimum of w_M . Notice that the figure shows a case in which the minimum wage is actually binding. This is why $w_M > w_j^*$ and, as a result, $\underline{x}_M > x_j^*$. As discussed in section 2.4, there are now two new crucial reservation values: \bar{x}_M , which is the productivity value at which firms would optimally pay the minimum wage; and \underline{x}_M , which is the productivity value at which firms make zero profit if they pay the minimum wage. Since profits are increasing in x , this generates a very intuitive optimal decision rule: no matches are made below \underline{x}_M , matches paying exactly the minimum wage are made between \underline{x}_M and \bar{x}_M , and matches paying the same wage as in the pre-policy environment are made above \bar{x}_M . Matches paying exactly the minimum wage are realized between \underline{x}_M and \bar{x}_M because workers are willing to accept that wage for the job (in fact, they would be willing to accept a lower wage for the job, i.e. the wage determined by the $w_1(x)$ schedule) and firms are still making positive profits on those jobs, even if they are forced to pay a higher wage than is optimal (recall that firms are making exactly zero profit at $x = \underline{x}_M$ and profits are increasing in x).

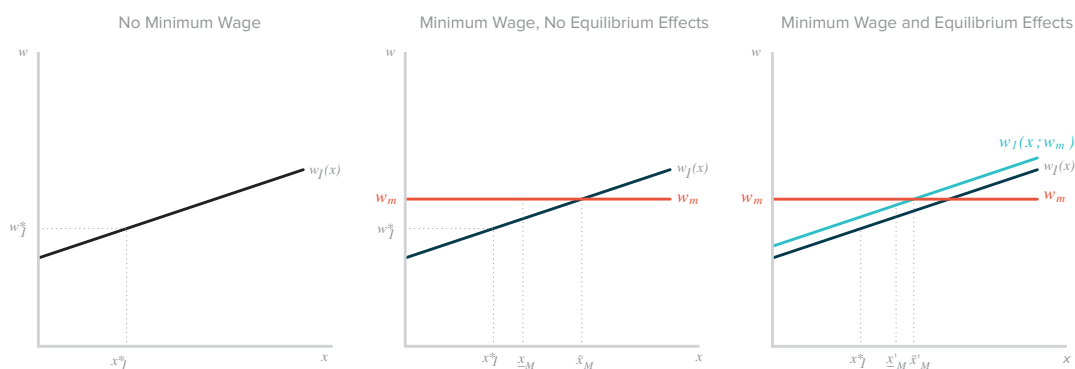
This environment can replicate the main features of the wage distribution in a labour market with a mandatory minimum wage: a positive proportion of workers earning exactly the minimum wage together with workers earning approximately a continuous distribution of wages above that. Notice, however, that $\underline{x}_M > x_j^*$. The level of employment is therefore **lower** than in the pre-policy environment. In this respect, the conclusion of this middle panel is similar to the result reported in figure 2 for the standard competitive model: the level of employment is lower, but employed workers earn higher wages.

The bottom panel in figure 3 adds the final element. If workers will earn a minimum wage higher than the w_j^* of the pre-policy environment, then they will be pickier in their search

¹⁴ See the equation in Flinn (2006). Equation 12 is the corresponding equation in this model with informality and social protection benefits.

behaviour. This is formally proven by showing the value of participating in the labour market when the minimum wage is present and higher than when this is not the case. In the notation of section 2.4, it is possible to prove that $U(w_M) > U$. As a result, workers have a higher outside option and therefore a higher bargaining power when bargaining with employers over wages. The wage schedule will then ‘shift up’ with respect to the pre-policy environment. This change is represented by the post-policy wage equation $w_1(x; w_m)$, which is a parallel upward shift of the pre-policy wage equation $w_1(x)$. In examining the corresponding equation in this extended model, it is clear why this movement occurs. In equation 12, the value of the outside option U is part of the intercept and therefore $U(w_M) > U$ implies a constant higher wage at each productivity value. This additional channel resulting from the equilibrium dynamic magnifies the employment effects discussed above. Wages are higher, not only for workers in the productivity range that pays the minimum wage but also for those at any higher point. In addition, the proportion of workers earning the minimum wage is smaller than in the intermediate case, since workers in the productivity range from \bar{x}_M to \bar{x}'_M earn the minimum wage in the intermediate case but earn more when taking into account equilibrium effects. Employment is lower, not only by the amount implied by the difference from \underline{x}_M to \underline{x}'_M but by the larger amount implied by the difference $(\bar{x}'_M - \bar{x}^*)$. It is important to emphasize this result. Although frictions imply a departure from the competitive model and generate rents, a binding minimum wage still generates negative employment effects. These negative effects are stronger once equilibrium effects are taken into account. The same is true for the positive effects on wages. A binding minimum wage increases wages, and this positive effect is stronger once equilibrium effects are taken into account. Finally, it is relevant to point out an implication of the model with frictions that was impossible to obtain in the competitive model of figure 2. When the labour market dynamic is taken into account, the **same** worker potentially pays the costs and receives the benefits of the minimum wage policy. A worker participating in the market will now spend more time in unemployment (due to the negative impact of the minimum wage on employment levels), but this is optimal for the worker because the expected wage offers will be higher (due to the positive impact of the minimum wage on wage levels). Wages are higher, either because the worker is guaranteed at least the minimum wage w_m at low productivity levels or because he or she will receive the higher wage implied by $w_1(x; w_m) > w_1(x)$ at higher productivity levels.

Figure 3: Minimum wage in a model with frictions and no informality



Source: Author.

2.5.3. The search friction model with informality

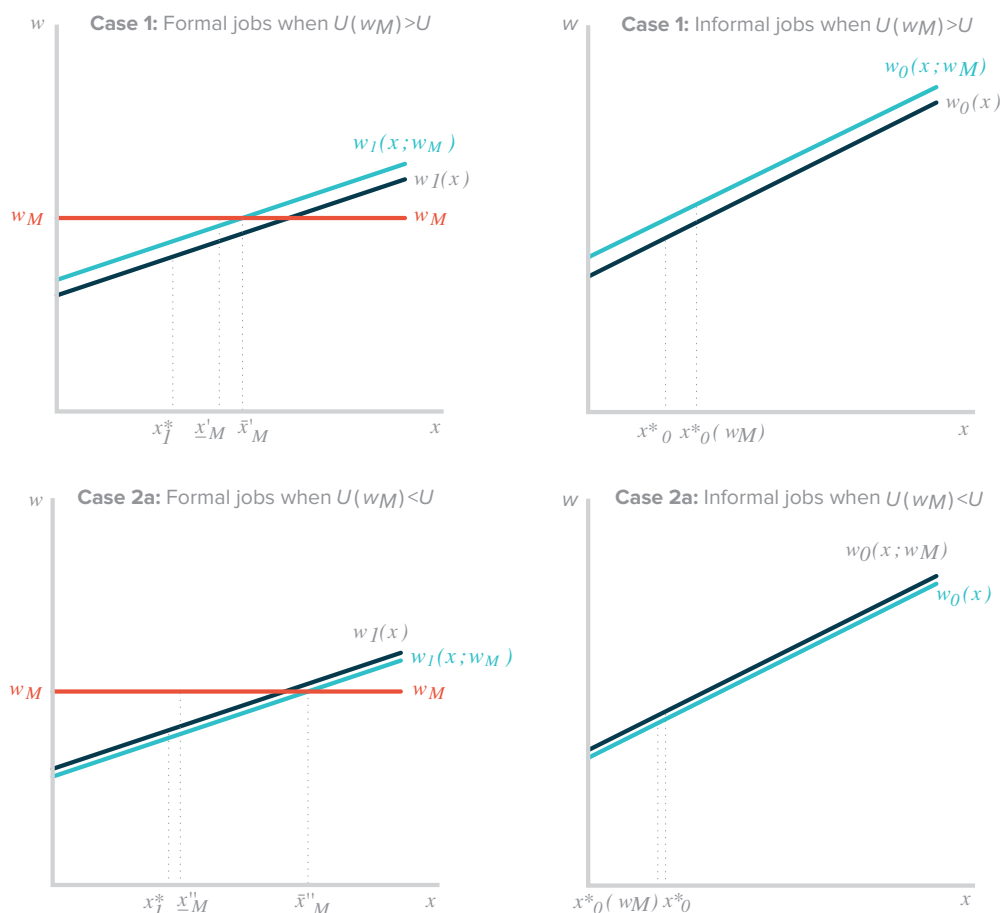
Figure 4 shows how these equilibrium impacts change when informality and related institutions are accounted for in the model, i.e. the environment presented in section 2.1. The left column demonstrates the impact on formal wages while the right column demonstrates the impact on informal wages. The primary difference in the model with informality is that the minimum wage is a binding constraint only in formal jobs and, specifically, only when bargaining for wages in formal jobs. Ignoring the equilibrium effects, the impact of the minimum wage is straightforward and corresponds to the intermediate case in figure 3, in which formal employment decreases, a positive proportion of workers in formal jobs earn the minimum wage and the remaining workers in formal jobs earn wages governed by the equation $w_f(x)$. Taking into account equilibrium effects, the result may be very different, even the opposite. First, consider a case that is similar to what has been discussed above, labelled as case 1.

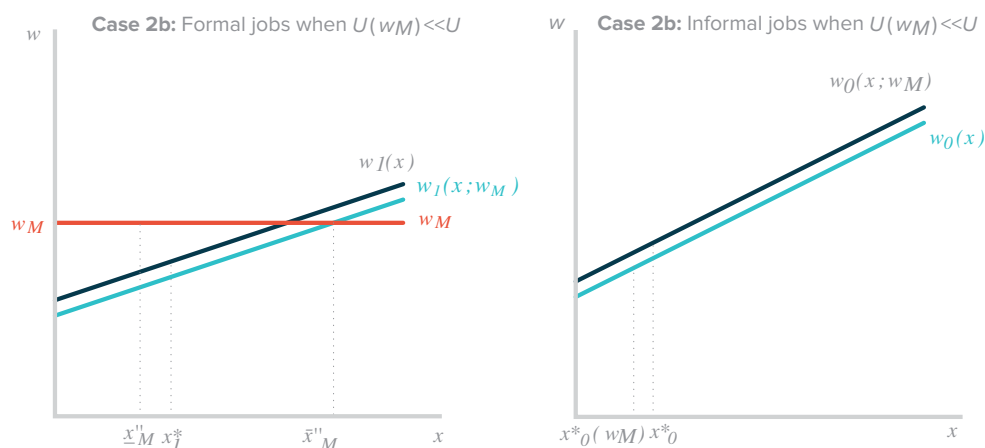
Case 1 is presented in the top panel of figure 4 and represents a situation in which the introduction of the minimum wage results in a net gain in the value of participating in the labour market, i.e. a situation where $U(w_M) > U$. As a result, the reservation productivity value to accept a formal job is higher than without minimum wage ($\underline{x}_M^f > x_j^*$), leading to a decrease in formal employment. At the same time, the proportion of workers in informal employment also decreases, since the reservation productivity value to accept a formal job is also higher. When workers and firms bargain over the wage in an informal job, they do not comply with the minimum wage regulation; however, they still take into account that formal jobs pay more owing to the policy. This expectation increases the workers' bargaining power in informal jobs, leading to higher wages at the same productivity and a decrease in informal employment. The standard employment effects of the minimum wage are therefore not only limited to formal employment but spill over to informal employment as well. To formally demonstrate this concept, recall that equation 10 shows that the reservation productivity value x_0^* is increasing in the value of the outside option. $U(w_M) > U$ then implies $x_0^*(w_M) > x_0^*$ which implies lower informal employment. In addition, equation 8 shows that the informal wage $w_0(x)$ is also increasing in the value of the outside option. $U(w_M) > U$ then implies higher informal wages at each point x of the productivity distribution $G_0(x)$. Negative employment effects are also present in self-employment but without any advantage in terms of income. The reservation value y^* is increasing in the value of unemployment U since the expectation of higher wages is taken into account by any searcher, even those that are presented with a self-employment offer. However, no positive increase in income is generated, since the self-employment productivity distribution is independent from the minimum wage. Equation 11 shows that y^* is increasing in U , leading to a reduction in self-employment. The lack of bargaining to determine the self-employment income y means that no increase in self-employment income is occurring as a result of the increased bargaining power for workers.

Case 2 is reported in the bottom two rows of panels in figure 4 and represents a situation in which the introduction of the minimum wage results in a net loss in the value of participating in the labour market, i.e. a situation where $U(w_M) < U$. If either case 1 or 2 arises in equilibrium, it is a function of the structure of the labour market as captured by the model's parameters. The following can be intuited. Recall once again the intermediate case in figure 3. Ignoring equilibrium effects, the introduction of a binding minimum wage **unambiguously** increases the reservation wage to accept a formal job ($\underline{x}_M^f > x_j^*$) and **unambiguously** leads to a loss of formal employment. It is a trade-off: a loss of employment in exchange for higher wages in the lower range of the productivity distribution. The net result of this trade-off may be positive or negative when equilibrium effects are taken into account. In other words, a searcher

in a labour market with a binding minimum wage is expected to take longer before finding an acceptable formal job and is expected to receive a higher wage when formally employed at relatively low productivity. The first effect decreases the value of participating in the market (i.e. leads to $U(w_M) < U$); the second effect increases it (i.e. leads to $U(w_M) > U$). The net effect may be positive (as in case 1) or negative (as in case 2). If the effect is negative, the impact on informal employment and self-employment is exactly the opposite of that described in case 1. Since $U(w_M) < U$, the reservation values to accept an informal or a self-employment job are lower, leading to greater employment in both states. In addition, the wage schedule for informal workers 'shifts down', leading to lower wages across the entire acceptable productivity distribution. The same shift occurs in the wage schedule for formal workers and may be so severe as to actually lower the reservation productivity value ($x_M^I < x_I^*$), leading to an increase in formal employment as well. This is the case represented in the final panels of figure 4, labelled case 2b. If this is not the case, the reservation productivity value remains higher than in the pre-policy environment ($x_M^I > x_I^*$) and formal employment will experience a net loss. Self-employment will always experience an increase in employment with no impact on earnings. To summarize, in case 2a, there is a net transfer of employment from the formal sector to the informal sectors; in case 2b, this may or may not be the case. In both case 2a and 2b, unemployment decreases, wages for informal salaried workers decrease, wages for formal salaried workers increase only for low productivity values and self-employment earnings remain unaffected.

Figure 4: Minimum wage in a model with frictions and informality





Source: Author.

A relevant policy question concerns which of the two cases an economy could be experiencing. Based on the implications from the model, the answer is not very strong: the economy could be in either of the two cases depending on the parameters. An answer can be provided only if the parameters of the model are estimated from data extracted from the economy of interest. Without this information, some broad empirical patterns may suggest that one case is more probable than the other. For example, if the contact rates between workers and firms are fixed, a larger formal sector should generate an impact closer to the one observed in an economy without informality. Recalling figure 3, this would be the same as case 1, where $U(w_M) > U$. As mention in section 2.2, with regard to equation 1, even this simple implication may not hold if the contact rates are endogenous since it becomes a function of the parameters.

3. Policy implications

3.1. Summary of labour market impacts

As a result of the discussion above, it is useful to distinguish two cases to study the impact of a mandatory minimum wage on a labour market with search frictions and informality. In the first case, the positive impact of the minimum wage on wage distributions is high enough to compensate for the loss of employment, leading to $U(w_M) > U$ (case 1 in figure 4). In the second case, the opposite is true, leading to $U(w_M) < U$ (case 2a and 2b in figure 4).

To simplify the discussion of policy implications, the main results are also summarized in table 2. The table contains four columns labelling the different effects that may occur with the introduction of a minimum wage. The first is the **direct effect**. This is the simple wage effect that is also present in the static model: no wage lower than the mandatory minimum wage can be offered. The second is labelled **equilibrium effect** and results from bargaining. Since the outside option affects the bargaining position of each agent, if the minimum wage improves the workers' bargaining position, they are able to receive higher wages at the same productivity, even at wage levels **above** the minimum wage. The third is labelled **selection effect** and results from the ability of workers and firms to accept or reject a match under

optimal conditions. This ability creates a difference between offered wages and accepted wages. Offered wages are those offered every time a worker and a firm meet. Accepted wages include only the subset of wages that are accepted because they are higher than the reservation wage. Accepted wages are therefore a **selection** of the offered wage. The presence of a minimum wage may change the reservation wage and the reservation productivity, amplifying or reducing the difference between offered wages and accepted wages. For example, a significant increase in observed wages could be seen because workers are pickier in accepting wage offers once the minimum wage is introduced, even if the increase in offered wages was not significant. The fourth and final column reports the **total effect**, summarizing the overall impact of introducing a mandatory minimum wage.

Case 1: $U(w_M) > U$:

- **Employment effects:** The proportion of workers in unemployment increases with respect to a case without minimum wage. The decrease in employment affects all labour market states, i.e. employment decreases in formal employment, informal employment and self-employment. However, the proportion of formal employment experiences the greatest decrease. As a result, the formality rate decreases. As reported in the second column of the top panel in table 2, employment for informal salaried workers or self-employed individuals decreases only through equilibrium effects, i.e. when workers are pickier when accepting jobs.
- **Labour income effects:** There is a positive proportion of formal salaried workers earning exactly the minimum wage. They are in jobs with relatively low productivity and would have accepted working for a lower wage in the pre-policy environment. For these workers, the minimum wage implies a direct wage increase. This effect is reported in the first column of the top panel in table 2 in the row denoted 'low productivity'. For relatively high productivity workers, i.e. those not earning the minimum wage, there is no direct effect on wages, but there is a positive equilibrium effect implied by bargaining. The same effect impacts all wages for informal salaried workers, both above and below the minimum wage. The labour income of self-employed individuals is unaffected by either the direct effect or the equilibrium effect, but it is positively affected by the selection effect. The selection effect is positive for all employed individuals, since the fact that workers are pickier when accepting jobs means they will accept only job offers that pay relatively more than in the pre-policy environment.

Case 2: $U(w_M) < U$:

- **Employment effects:** The proportion of workers in unemployment decreases with respect to a case without minimum wage; however, which labour market state gains employment is heterogenous. The proportions of informal salaried workers and self-employed workers **unambiguously** increase. The only employment impact of the minimum wage on these states is in the equilibrium effect. Since the value of participating in the market is lower, the reservation values are lower, which leads workers to accept job offers more easily. This effect is reported by the upward arrows in the second column of the middle and bottom panels of table 2, in the rows entitled 'informal salaried employment' and 'self-employment'. In contrast, the proportion of formal salaried workers may increase or decrease as a result of the equilibrium effects impacting the wage schedule. If the equilibrium effects are not too strong, the reservation productivity value to accept a formal job is higher than in the pre-policy environment, leading to a decrease in formal employment (case 2a in figure 4 and table 2). If the equilibrium effects are strong enough, the reservation productivity value to accept a formal job is lower than in the pre-policy

environment, leading to an overall increase in formal employment (case 2b in figure 4 and table 2). Even in this second case, however, it is highly unlikely that the equilibrium effects could be high enough to increase the formality rate.

- Labour income effects:** As in case 1, there is a positive proportion of formal salaried workers earning exactly the minimum wage. These are relatively low productivity workers who earn a higher wage than in the pre-policy environment. All other salaried workers (formal workers earning above the minimum wage and all informal workers) experience a wage drop due to the equilibrium effect and bargaining. Workers have a lower outside option and will receive a lower wage at the same productivity x . Self-employment labour income remains unaffected; however, contrary to case 1, the average earned self-employment income decreases due to the selection effect. Workers are less picky and accept self-employment opportunities at lower y values.

As this discussion makes clear, the introduction of the minimum wage generates both benefits and costs for workers. While some salaried workers will experience higher wages, the relative increase varies for different workers. In some cases, the wage increase is limited to the very narrow set of minimum wage earners; in other instances, the wage increase involves the entire set of salaried workers. These benefits are coupled with employment effects: negative employment effects are present in most, albeit not all, cases. Negative employment effects are generally more intense for formal salaried workers. Employment effects can be read in the dynamic model developed here as time in the unemployed (searching) state. Negative employment effects translate not only to a higher unemployment rate in a steady state but also to a longer unemployment duration. Even with the simple utility function adopted here, these effects have different and sometimes opposite impacts on workers' welfare. As a result, it is not possible to simply examine earned wages or aggregate employment rates to determine whether a worker is better off in the post-policy environment. A complete characterization of workers' welfare requires a more sophisticated outcome variable that is able to summarize both employment and wage effects. The next section proposes such a measure.

Table 2: Impacts of a mandatory and binding minimum wage

Labour market outcomes:	Direct effect	Equilibrium effect	Selection effect	Total effect
Case 1: $U(w_M) > U$				
Employment:				
Formal salaried employment	↓	↑	..	↓
Informal salaried employment	=	↓	..	↓
Self-employment	=	↓	..	↓
Formality rate	↓	↑	..	↓
Unemployment rate	↑	↓↑	..	↑
Income:				
Formal wages				
Low productivity	↑	=	↑	↑
High productivity	=	↑	↑	↑
Informal wages	=	↑	↑	↑
Self-employment income	=	=	↑	↑
Case 2a: $U(w_M) < U$				
Employment:				
Formal salaried employment	↓	↑	..	↓
Informal salaried employment	=	↑	..	↑
Self-employment	=	↑	..	↑
Formality rate	↓	↑	..	↓
Unemployment rate	↑	↓	..	↓

Income:				
Formal wages				
Low productivity	↑	=	↑	↑
High productivity	=	↓	↑	↓↑
Informal wages	=	↓	↓	↓
Self-employment income	=	=	↓	↓
Case 2b: $U(w_M) \ll U$				
Employment:				
Formal salaried employment	↓	↑	..	↑
Informal salaried employment	=	↑	..	↑
Self-employment	=	↑	..	↑
Formality rate	↓	↓↑	..	↓
Unemployment rate	↑	↓	..	↓
Income:				
Formal wages				
Low productivity	↑	=	↑	↑
High productivity	=	↓	↑	↓↑
Informal wages	=	↓	↓	↓
Self-employment income	=	=	↓	↓

Source: Author.

Note: $U(w_M)$ and U denote the value of participating in a labour market with or without a minimum wage, respectively. See section 3.1 for additional comments and definitions. The up arrow (↑) denotes an increase in variable of interest, the down arrow (↓) denotes a decrease, the equal sign (=) denotes no change and the two dots (..) mean not applicable.

3.2. Impact of the minimum wage on welfare

As the policy discussion has made clear, the introduction of a mandatory minimum wage creates winners and losers. In the standard static model discussed in section 2.5.1, the winners and losers are straightforward: the employed win, since they receive a higher wage, and the involuntary unemployed lose. When dynamic considerations are taken into account, the implications are more nuanced.

Remaining as close as possible to the model presented in section 2, each agent has the possibility to move through all possible labour market states. For example, an agent may search for a job while unemployed for a time, then transition to a minimum wage job and, following another search, find a job at a higher wage. He or she may then begin the cycle again. What becomes crucial then is not only the ‘value’ or utility of each labour market state but also the duration an agent is expected to spend in it. The present discounted value of participating in the market is a very simple welfare measure that is able to account for all of these aspects since it includes the expectations of future shocks and the optimal decision rules implemented as a reaction to those shocks. Notice that the frequency of shocks is precisely the information relevant to assessing the duration and transitions of the probabilities mentioned above. In conclusion, the value of unemployment denoted by $U(w_M)$ is the appropriate measure to evaluate the *ex ante* welfare of a given individual participating in a labour market with minimum wage w_M . This is the welfare an agent may expect while still searching for a job, i.e. **before** meeting an actual employer, potentially accepting a job offer and beginning to earn an income. The welfare impact of introducing a mandatory and binding minimum wage is then directly derived by the discussion of figure 4 and table 2. Case 1 characterizes a situation in which the minimum wage increases welfare, while case 2 characterizes a situation in which the minimum wage decreases welfare. Notice that in case 1, welfare increases despite the unemployment rate increasing and the formality rate decreasing, i.e. despite the fact that two variables frequently targeted by policy interventions move in the opposite direction of

what is typically thought to increase workers' welfare. Another relevant remark is that the same labour market may move from case 1 to case 2 for different levels of minimum wage. This opens the door to the possibility of an 'optimal' level of minimum wage, where optimality is reached when workers' welfare is maximized.

This welfare characterization is quite neat and economical but appears to contain an important limitation: it only considers workers' welfare and not firms' welfare. While the model does not provide a full characterization of the demand side of the market, the free-entry condition mentioned in section 2.3 allows for a partial solution. Under free entry, the expected profit of entering the market is always zero. However, the expected profit of entering the market is exactly the present discounted value of participating in the market from the point of view of the firm, i.e. the streams of profits and vacancy costs accrued by reacting optimally to shocks. In this sense, the welfare measure in *ex ante* terms should only consider workers, since the firms' welfare value in *ex ante* terms is always zero. As a result, $U(w_M)$ can be seen as the appropriate measure of **overall** welfare under the free-entry assumption.

This is only a partial solution because the firms' behaviour, which guarantees zero *ex ante* profit, may also impact some structural parameters of the model. Meeting rates are most studied in the literature. If the number of firms entering the market increases or decreases, then the arrival rate of jobs to workers, which is denoted with λ , may be affected.¹⁵ This will introduce another equilibrium channel in addition to the outside option effect previously discussed. The elasticities of these impacts vary and are a function of the size of the minimum wage changes with respect to the baseline. Flinn (2006) estimates a search model with minimum wage and bargaining but no informality on US data and finds that the impact of changes in the minimum wage on arrival rates begins to become significant for relatively modest changes. To provide a sense of the order of magnitude, he estimates an optimal minimum wage of US\$8.66 per hour when ignoring firms' impacts on meeting rates and \$3.36 per hour when taking them into account. At the time, the federal minimum wage in the United States was between \$4.25 and \$4.75 per hour.

Four final remarks are important to better assess the policy implications of this model. First, the optimal minimum wage level depends on the outcome variable of interest. Specifically, the optimal minimum wage when policymakers want to maximize welfare may differ significantly from the optimal minimum wage when policymakers want to maximize gross domestic product. Second, the existence of a positive optimal minimum wage does not mean that imposing a binding minimum wage is always optimal or that any increase in a binding minimum wage is optimal. In fact, optimality implies that there is always a point above which the minimum wage is too high. Depending on the characteristics of the economy, the optimal policy may be an increase or a decrease in the minimum wage. As a result, each country policy should be evaluated on a case-by-case basis. Third, the welfare measure used in the discussion $U(w_M)$ has a solid theoretical foundation but also has significant limitations. As mentioned above, it relies on the strong assumption of free entry of firms, which implies that firms' *ex ante* welfare is always zero; therefore, their welfare considerations have a limited impact on the outcome variable of interest. Within this same modelling framework, there is an alternative welfare outcome variable that can account for firms' welfare more directly: an *ex post* welfare measure focusing on the steady-state flow welfare.¹⁶ Implementing policy evalua-

¹⁵ A common way to formalize this impact is the matching function. See the review by Petrongolo and Pissarides (2001).

¹⁶ For applications of *ex post* welfare measures in the presence of a binding minimum wage, see Flinn and Mullins

tions based on this welfare variable is theoretically straightforward because it requires more data in order to estimate the demand side of the labour market as well. Fourth, while this paper focuses on minimum wage policies, many other policies can be studied in the context of this model, some of which can reach similar objectives. These policies are appropriate in any labour market affected by frictions and distortions that render the perfectly competitive model a bad approximation. A typical example is the reduction of search frictions. Examples of optimal policies that increase both welfare and gross domestic product include favouring meetings of workers and firms, addressing the lack of information about job availability and decreasing reallocation costs in job-to-job transitions.

4. Conclusion

The body of literature on the minimum wage is vast, reflecting the importance of this policy instrument in many labour markets around the world. This relevance extends to labour markets in Latin America, where both increases and decreases in the levels of the minimum wage have been linked to significant fluctuations in wage inequality.¹⁷ The contribution of this background paper is to demonstrate how small departures from the standard static framework typically used to inform policies and empirical works may generate vastly different conclusions about the impact of minimum wage policies on workers' welfare. In particular, the combination of a binding minimum wage in the formal market and a large informal sector in the economy suggest that the introduction of a minimum wage may be both welfare increasing and welfare decreasing.

To account for dynamic considerations in a tractable way, this paper extends the well-known search and matching framework by introducing both a binding minimum wage and a large proportion of workers hired informally.¹⁸ This framework, which is arguably more complete, identifies three channels that transmit minimum wage policies to labour market outcomes. The first is labelled **direct effect**. This is the simple wage effect that is also present in the static model: no wage lower than the mandatory minimum wage can be offered, introducing a price floor for labour and decreasing its demand. The second is labelled **equilibrium effect** and results from bargaining. The workers' bargaining position with respect to firms is affected by the presence of the minimum wage. Since labour market frictions generate rents to be shared between workers and firms, a minimum wage that improves the workers' bargaining position allows them to receive a higher share of the rents generated by the job. Interestingly, this channel is present both when bargaining for formal jobs that follow minimum wage rules and for informal jobs that do not. This channel also implies that the minimum wage even increases wages at wage levels **above** the minimum wage. The third channel is labelled **selection effect** and results from the ability of workers and firms to accept or reject a job under optimal conditions. The presence of a minimum wage may change the reservation wage and the reservation productivity (the points of indifference between accepting or rejecting a job), amplifying or

(2015). For an application in Latin America but without a binding minimum wage, see Bobba et al. (2017).

¹⁷ See, for example, Bosch and Manacorda (2010) for Mexico and Engbom and Moser (2018) for Brazil.

¹⁸ For reviews of the search and matching framework, see, for example, Eckstein and van den Berg (2007) and Rogerson et al. (2005). Contributions studying minimum wage policies within the search and matching framework include Eckstein et al. (2011), Flinn (2006, 2011) and van den Berg and Ridder (1998). Contributions studying informality within the search and matching framework include Bobba et al. (2017), Bosch and Esteban-Pretel (2012) and Meghir et al. (2015).

reducing the difference between offered wages and accepted wages. This effect is particularly relevant for empirical works, as only accepted wages are typically observed in the data.

Given the richness of channels and the potential for ambiguous effects, there is room to study optimal policy. Unlike in the standard static competitive model of the labour market, the optimal minimum wage in an economy with frictions, informality and bargaining may be significantly different from zero. At the same time, an increase in already high levels of minimum wages may decrease welfare significantly. To determine the optimal minimum wage of a given country, an estimated version of the model proposed in this paper is needed. If sophisticated estimations of a similar model for some Latin American countries are now present in the literature,¹⁹ none include the presence of a mandatory minimum wage. This suggests a promising direction for policy-relevant future research on Latin American labour markets.

¹⁹ See Meghir et al. (2015) for Brazil and Bobba et al. (2017, 2020) for Mexico.

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